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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed six manuscript pages and three figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

New Computational Method to Determine Trim Angle of Attack for Reentry Vehicles

Guo Zheng,* Liu Jun,† and Qu Zhang-hua†
National University of Defense Technology, 410073 Changsha, People's Republic of China

Introduction

A LARGE majority of modern advanced reentry vehicles adopt the lifting reentry style, so that a large lateral acceleration capability is obtained. This lateral maneuver ability and landing accuracy depends deeply on appropriate trim lift-to-drag ratio, that is, suitable trim angle of attack (AOA) once the vehicle's shape contour has been carefully designed. Therefore, accurately determining the trim AOA of a reentry vehicle is important for its design. Usually, trim AOA can be obtained by experiment methods including force-measuring method, wind-tunnel flight testing and freely oscillating method. In force-measuring method, the aerodynamic forces of the vehicle at different AOA are measured, and then the data are interpolated to find the trim AOA at which the pitching moment about center of gravity is zero. Because the pitch moment near trim AOA is very small and varies smoothly, it is difficult to be accurately measured, so that the force-measuring method often introduces a lot of error into the result.[1] Wind-tunnel flight testing usually uses a much smaller model; it is also very difficult to ensure the offset of center of gravity in such small model. Luo and Bi[1] had determined the trim AOA of a reentry capsule using a freely oscillating method in a supersonic wind tunnel, but this method was conditioned by the accuracy of angle measurement. The computational-fluid-dynamics (CFD) method currently used widely to determine trim AOA is similar to the force-measuring method in wind tunnels, which requires many steady-state calculations. A new freely oscillating method via numerical calculation has been developed in this note, which uses dynamic-unstructured-grid technique (DUT) and obtains the trim AOA directly by single computation. In this new method, the vehicle is allowed to oscillate about an axis perpendicular to its symmetry plane and passing through its center of gravity under the function of aerodynamic forces. The oscillating unsteady flow simulation was started from the steady-state solution at arbitrary initial AOA, and ended when the AOA oscillation was damped to close enough to trim condition. As the dynamic response property of the vehicle did not need to be considered, to make the calculation more efficient relatively small inertia and large damping moment were chosen artificially. The DUT used here provides a high grid-deforming ability,[2] so that the initial AOA can be chosen within an adequate wide range. As a demonstration, the trim AOA of reentry capsule was determined; the calculation results agree well with experiment data. This new method has the potential to handle complex geometries and has relatively high efficiency and can be widely used for generic spacecraft.

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*Associate Professor, College of Aerospace and Material Engineering.
†Professor and Chair, College of Aerospace and Material Engineering.

Dynamic Unstructured Grids

DUT is generally implemented by the combination of grid deforming and local remeshing, but just using grid deforming is sufficient for cases with relatively small boundary displacement such as the problem discussed in this Note. Consequently the computational efficiency is improved, and the interpolation induced diffusion loss is avoided.

The model used to control mesh deforming is improved spring analogy with boundary improvement[3] and torsional effect improvement,[3] which has been demonstrated to work well.[2] The whole three-dimensional unstructured mesh is considered as a system of interconnected springs by regarding each edge of each cell as a tension spring. The spring stiffness is written as

$$K_{ij} = (\phi/\beta)(l_{ij})^{\psi} \tag{1}$$

Here ϕ is stiffening factor to increase the spring stiffness near the moving boundary; β denotes torsional effect and is defined as the smallest internal angle (unit in radian) among cell face internal angles facing the edge; and l_{ij} is the length of the edge connecting nodes i and j. In the present calculation, the parameter ϕ and exponent ψ are chosen as $\phi = 5$ and $\psi = -2$ for one element layer adjacent to the moving boundary, while $\phi = 1$ and $\psi = -2$ are for the interior of the mesh. As the mesh deforms, the tensile force exerting on node i is assumed always equal to the initial value computed so as to maintain the initial grid in the absence of any displacement, and can be expressed as

$$\boldsymbol{f}_i = \sum_{j=1}^{N_i} K_{ij}(\boldsymbol{x}_j - \boldsymbol{x}_i) \tag{2}$$

where $\boldsymbol{x}_i$ is the position vector of node i and N_i denotes the number of neighbor nodes connecting with node i. For all nodes Eq. (2) can be expressed as a linear system $[\boldsymbol{A}]\{\boldsymbol{x}\} = \{\boldsymbol{b}\}$, where $[\boldsymbol{A}]$ is the coefficient matrix formed by spring stiffness and is diagonal dominant. The coordinates of moving grid points can be updated by solving the preceding linear system using several Jacobi iterations. Because the coordinates of fixed boundary points and driving boundary points are known, the boundary condition of the spring system is Dirichlet type. A satisfactory accuracy can be obtained after three or four Jacobi iterations.

From Eq. (1) one can see that the stiffness becomes a function of node coordinates; this fact introduces nonlinear effect to the spring system. To avoid divergence of the iterative solution, the frequency of stiffness updating must not to be too high.

Numerical Discretization

Governing Equations

Three-dimensional time-dependent compressible Euler equations in arbitrary Lagrangian–Eulerian (ALE) finite volume description can be expressed for a bounded domain Ω with a boundary $\partial\Omega$ as follows:

$$\frac{\partial}{\partial t}\iiint_{\Omega} \boldsymbol{Q}\,\mathrm{d}V + \iint_{\partial\Omega} \boldsymbol{F}(\boldsymbol{Q})\cdot n\,\mathrm{d}S = 0 \tag{3}$$

where $\boldsymbol{n}$ denotes the unit vector pointing out normal to the control volume boundary. The vector of conserved variables $\boldsymbol{Q}$ and the convective fluxes are given by

$$\boldsymbol{Q} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{pmatrix}, \qquad \boldsymbol{F}(\boldsymbol{Q})\cdot\boldsymbol{n} = (\boldsymbol{U}\cdot\boldsymbol{n})\begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e + p \end{pmatrix} + p\begin{pmatrix} 0 \\ n_x \\ n_y \\ n_z \\ a_t \end{pmatrix}$$

Here ρ is density; u, v, and w are components of fluid velocity; p is pressure; e denotes the total energy per unit volume; $\boldsymbol{U}$ represents the contravariant velocity; a_t is the normal velocity of control volume boundary motion; n_x, n_y, n_z are components of vector $\boldsymbol{n}\cdot\boldsymbol{U}$ and a_t can be written as

$$\boldsymbol{U} = \{(u - x_t), (v - y_t), (w - z_t)\} \tag{4}$$

$$a_t = x_t n_x + y_t n_y + z_t n_z \tag{5}$$

where x_t, y_t, z_t are components of grid velocity a_t.

Discretization Scheme

Integral of Eq. (3) in control volume (grid cell) can be written as

$$\left(V^{m+1}\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t} + \boldsymbol{Q}^m\frac{\mathrm{d}V}{\mathrm{d}t}\right) = -\sum_{k=1}^{4}\boldsymbol{F}_k\cdot\boldsymbol{S}_k \tag{6}$$

In this equation m denotes time step, and $\boldsymbol{S}_k$ represents the vector area of face k of the cell. A second-order approximation of conserved variables on cell face is obtained by expanding the cell-centered solution with a Taylor series. The Riemann problem on cell face is solved by van Leer or Steger flux-vector splitting method, that is,

$$\boldsymbol{F}_k = \boldsymbol{F}_k^{+}(\boldsymbol{Q}_L) + \boldsymbol{F}_k^{-}(\boldsymbol{Q}_R) \tag{7}$$

where the reconstructed conserved variables are calculated as

$$\boldsymbol{Q}_L = \boldsymbol{Q}_i + \phi_{ik}(\nabla\boldsymbol{Q})_i\cdot\boldsymbol{r}_{ik}, \qquad \boldsymbol{Q}_R = \boldsymbol{Q}_j + \phi_{jk}(\nabla\boldsymbol{Q})_j\cdot\boldsymbol{r}_{jk} \tag{8}$$

Here ϕ is a flux limiter. This formulation requires that the solution gradient be known at the cell centers. A general approach to estimate the solution gradient is the Gauss–Green theorem, which requires a large memory occupation. In the present work a new scheme first proposed by Frink[4] based on geometrical invariant features of tetrahedra was used. Using this new scheme, the first one of Eq. (8) can be simplified as

$$\boldsymbol{Q}_L = \boldsymbol{Q}_i + \phi_{ik}\{(\boldsymbol{Q}_{p1} + \boldsymbol{Q}_{p2} + \boldsymbol{Q}_{p3})/3 - \boldsymbol{Q}_{p4}\}/4 \tag{9}$$

The definition of variables in Eqs. (8) and (9) is shown in Fig. 1. The nodal quantities are determined at each node by a weighted average of the surrounding cell-centered solution quantities. The average procedure has a somewhat smoothing effect, and so a less dissipative flux limiter was used. It can be written as follows:

$$\phi_{ik} = \phi\left(\frac{\boldsymbol{Q}_j - \boldsymbol{Q}_i}{\nabla\boldsymbol{Q}_i\cdot\boldsymbol{r}_{ik}}\right) \tag{10}$$

$$y\phi\left(\frac{x}{y}\right) = \Psi(x, y) \tag{11}$$

$$\Psi(x, y) = 0.5[\mathrm{sign}(x) + \mathrm{sign}(y)]\min(|x|, |y|) \tag{12}$$

Solutions are advanced in time by explicit four-stage Runge–Kutta integration with up to second-order temporal accuracy. To avoid grid motion induced error, the geometric conservation law (GCL) is satisfied numerically as described in Ref. 5.

Results and Discussion

Determination of the trim AOA for a reentry capsule with the developed new method is presented in this section. The computational grid consists of 19,970 nodes and 104,868 cells, and the surface grid is shown in Fig. 2. The present calculations were made for the condition of $M_\infty = 4.0$ and initial AOA as $\alpha_0 = 15$ deg. To make an assessment of the repeatability of the result, five computations were conducted with different inertia and artificial damping moment combination. The five cases are

1) $T_z = -0.01\omega$, $I_z = 0.1$

2) $T_z = -0.02\omega$, $I_z = 0.1$

3) $T_z = -0.03\omega$, $I_z = 0.1$

4) $T_z = -0.05\omega$, $I_z = 0.1$

5) $T_z = -0.03\omega$, $I_z = 0.05$

Here T_z denotes the artificial damping moment, which is assumed inversely proportional to angular velocity ω in reverse direction; I_z is the inertia of the capsule. All of them are nondimensionalized with freestream density and reference length. The criteria to judge whether the oscillation of the capsule is damped to trim state is

$$\left|\frac{C_m^n - C_m^{n-500}}{t^n - t^{n-500}}\right| \le 0.01, \qquad \left|C_m^n\right| \le 0.0001 \tag{13}$$

Here C_m represents pitching-moment coefficient, t denotes time, and n is the time step.

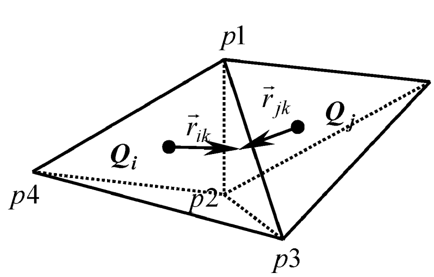

Fig. 1 Definition of variables.

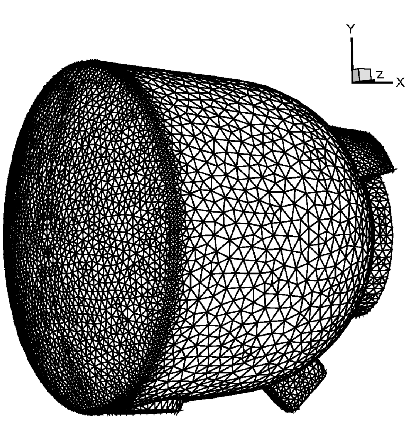

Fig. 2 Surface grid of the reentry capsule.

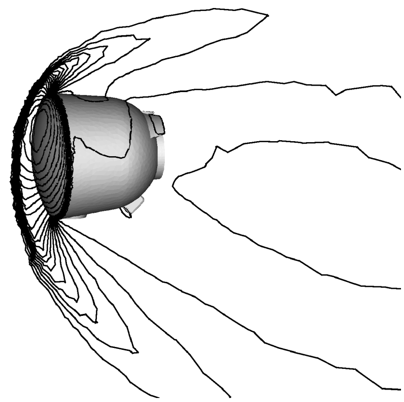

Fig. 3 Pressure contours at initial angle of attack.

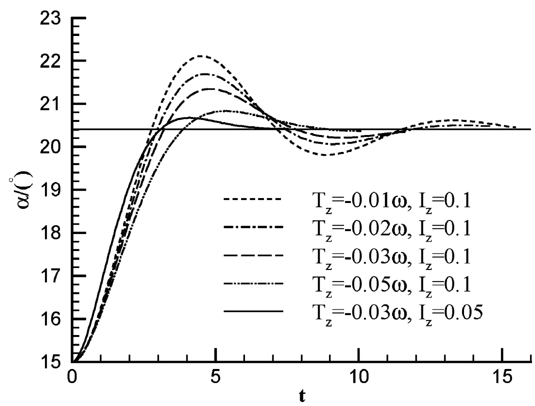

Fig. 4 Oscillation history of angle of attack.

The steady-state pressure contours are shown in Fig. 3, and the damping histories of AOA oscillation are presented in Fig. 4. Figure 4 indicates that the trim AOA calculated is 20.4 deg, and the results with different combination of inertia and artificial damping moment are well repeated, which demonstrates the robustness of the flow solver and dynamic grid algorithm. Here is the comparison of calculated results with experiment data quoted from Ref. 1: trim AOA α_T for the force-measuring method is 21.70 deg, for the freely oscillating method is 18.75, and for the calculation is 20.40. The calculated result falls between the two experimental results using different measurement techniques. So the accuracy of this new freely oscillating CFD method to directly determine trim AOA is validated. According to Fig. 4 and the preceding list, we think that the error of the result is mainly caused by the neglect of viscous effect, although viscosity has little effect on pitching aerodynamic property in supersonic flow of Mach number of 4.

Conclusions

A new method to directly determine the trim angle of attack (AOA) for spacecraft using dynamic unstructured grid technique was developed. Grid movement was controlled by improved spring analogy. Euler equations in arbitrary lagrangian–Eulerian description were solved using a MUSCL-type finite volume scheme. As a validation, the trim AOA of reentry capsule was correctly calculated. This new method has the potential to be applied for complex geometries caused by unstructured grid framework and has relatively high efficiency because single computation is needed.

Acknowledgments

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P. Huseman
Associate Editor